1.) You are given the following information:

(i) The sum of the present values of a payment of X at the end of 10 years and a payment of Y at the end of 20 years is equal to the present value of a payment of X + Y at the end of 15 years.

(ii) X + Y = 100

(iii) i = 5%

$\frac{X}{1.05^{10}} + \frac{Y}{1.05^{20}} = \frac{100}{1.05^{15}}$

43.93

Calculate X.

2.) An annuity pays 1 at the end of each year for n years. Using an annual effective interest rate of i, the accumulated value of the annuity at time $(n + 1)$ is 13.776. It is also known that $(1 + i)^n = 2.476$.

Calculate n.

$\left(\frac{2.476 - 1}{i}\right) \cdot (1+i) = 13.776$

$2.476i - i + 2.476 - 1 = 13.776i$

$i = 0.12$

$1\left(\frac{(1.12)^{n+1} - 1}{.12}\right) = 13.776$

$\left[\frac{\ln((.12 \cdot 13.776) + 1)}{\ln 1.12}\right] - 1 = n$

3.) Ron is saving for his retirement by making annual payments until he retires. Ron turned 20 years old today and will make deposits of $500 at the beginning of each year for the next 15 years. Just before Ron turns 35 he decides to increase his next payment to $4000. He continues making $4000 annual payments until his last payment at age 64. His bank account earns 6.5% interest effective annual.

Starting at age 65 Ron would like to withdraw a level payment amount at the beginning of each year for 20 years. What level payment amount will completely drain Ron's account after the last withdrawl?

4.) Peter deposits 400 into a bank account at time t = 0. During the first year, the bank credits interest at an annual effective rate of 10.25%.

Peter makes an additional deposit of 42 into his bank account at time t = 1. During the second year, the bank credits interest at a force of interest $\delta_t = \frac{1}{K+t}$.

The total amount in Peter's account at time t = 2 is 552.

Calculate K.

5.) Borat puts 10,000 into a bank account that pays an annual effective interest rate of 4% for 10 years. If a withdrawal is made during the first 5 and one-half years, a penalty of 5% of the withdrawal amount is made. Borat withdraws K at the end of each of years 4, 5, 6, and 7. The balance in the account at the end of year 10 is 10,000.

Calculate K.

6.) Your professor borrows \$1,000 from you at an annual effective rate of interest i. He agrees to pay back \$1,000 after six years and \$1,366.87 after another six years. Three years after his first payment to you, your professor gets a raise in his salary and wishes to repay the outstanding balance. What is the amount that he would now have to pay?

7.) Fund A is invested at an effective annual interest rate of 3%. Fund B is invested at an effective annual interest rate of 2.5%. At the end of 20 years, the total in the two funds is \$10,000. At the end of 31 years, the amount in fund A is twice the amount in Fund B.

Calculate the total in the two funds at the end of 10 years.

8.) Using an annual effective interest rate i, you are given:

(i) The present value of 2 at the end of each year for 2n years, plus an additional 1 at the end of each of the first n years, is 36.

(ii) The present value of an n-year deferred annuity-immediate paying 2 per year for n years is 6.

Calculate i.

9.) On September 30, 2006, Cory deposits X into a bank account. The account is credited with simple interest at the rate of 10% per year.

On the same date, Julie deposits X into a different bank account. The account is credited interest using a force of interest $\delta_t = \frac{2t}{(t^2+k)}$.

From the end of the 4th year until the end of the 8th year, both accounts earn the same dollar amount of interest.

Calculate k.

10.) You are the account manager at ABC Bank Incorporated. You were just told that somehow you lost part of the information on Joe Customer's account. Here is the information that you still have on the account:

- The account was opened exactly 3 years ago and the current balance is \$9,725.78
- Joe only made two deposits into the account, the first one was when the account opened and the second one was a year later.
- Joe's first deposit was for \$6,000
- The account credited interest as follows:
  - 7% simple interest during the 1st year
  - 8% constant force of interest during the 2nd year.
  - 9% compound discount during the 3rd year.

Calculate the amount of Joe's second deposit.

Term Test #1 Solutions

(1) $PV = Xv^{10} + Yv^{20} = (X+Y)v^{15}$

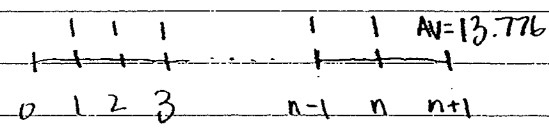
$(X+Y)v^{15} = \dfrac{100}{(1.05)^{15}} = 48.1017$ $\qquad Xv^{10} + Yv^{20} = \dfrac{X}{(1.05)^{10}} + \dfrac{Y}{1.05^{20}} =$

$.6139X + .3769Y = 48.1017$ $\qquad Y = \dfrac{48.1017 - .6139X}{.3769}$

$\Rightarrow X + Y = X + \left(\dfrac{48.1017 - .6139X}{.3769}\right) = 100 \Rightarrow .2370X = 10.41275$

$\boxed{X = 43.93}$

(2)



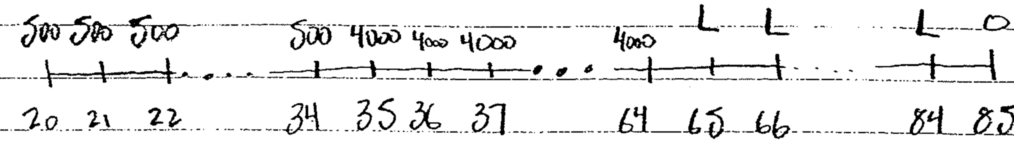
FV at time n: $1 \cdot s_{\overline{n}|i}$ $\qquad$ FV at time $n+1$: $1 \cdot s_{\overline{n}|i} \cdot (1+i)$

$\Rightarrow 1 \cdot \left[\dfrac{(1+i)^n - 1}{i}\right] \cdot (1+i) = 1 \cdot \left[\dfrac{2.476 - 1}{i}\right] \cdot (1+i) = 13.776$

$\Rightarrow \dfrac{1.476}{i} \cdot (1+i) = 13.776 \Rightarrow 1.476 + 1.476i = 13.776i$

$\boxed{i = 12\%}$

(3.)



FV at age 65 of deposits = PV at age 65 of withdrawls of L.

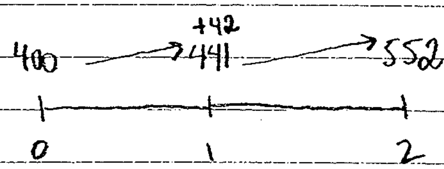
FV of deposits $= 500\,\ddot{s}_{\overline{15}|} \cdot (1+i)^{30} + 4000\,\ddot{s}_{\overline{65-35}|} =$

$500(25.7540)(6.6144) + 4000(91.9892) = 453130.18$

PV of withdrawls of size L : $L \cdot \ddot{a}_{\overline{20}|.065} = L \cdot (11.7347)$

$\Rightarrow 453{,}130.18 = L \cdot (11.7347) \Rightarrow \boxed{L = 38614.50}$

(4.)



$AV_1 = 400 \cdot (1.1025) = 441 \qquad AV_1{}' = 441 + 42 = 483$

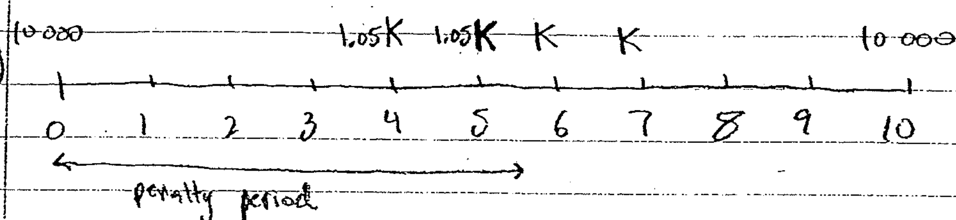
$AV_2 = 483 \cdot \exp\left[\int_1^2 \frac{1}{K+t}\,dt\right] = 552$

$= 483 \cdot \exp\left[\ln(K+t)\Big|_1^2\right] = 483 \cdot \exp\left[\ln(K+2) - \ln(K+1)\right]$

$= 483 \cdot \frac{(K+2)}{(K+1)} = 552 \Rightarrow \frac{K+2}{K+1} = 1.142857$

$\Rightarrow \boxed{K = 6}$

(5.)



*note: you get 1.05K since he withdraws K and is penalized an additional .05K

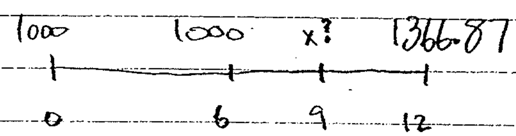
FV_{10} of deposit − FV_{10} of withdrawals = 10 000

FV of deposit = $10000(1.04)^{10} = 14\,802.44$

FV of withdrawals = $1.05K(1.04)^6 + 1.05K(1.04)^5 + K(1.04)^4 + K(1.04)^3$
$= 4.90 \cdot K$

$\Rightarrow 14\,802.44 - 4.90K = 10\,000$ $\boxed{K = 979.93}$

(6.)



$\Rightarrow 1000 = 1000v^6 + 1366.87v^{12}$

let $x = v^6$ $\Rightarrow 1366.87x^2 + 1000x - 1000 = 0$

quadratic gives $x = .56447$ $\Rightarrow v^6 = .56447 = \dfrac{1}{(1+i)^6}$ $\Rightarrow i = 10\%$

· At time 9 he still owes you 1366.87 at t=12, so if you discount this amount 3 years you get...

$PV_9 = \dfrac{1366.87}{1.10^3} = \boxed{1026.95}$

(7) Fund A: $a\cdot(1.03)^{20} = 1.8061\cdot a$ Fund B: $b\cdot(1.025)^{20} = 1.6386\cdot b$

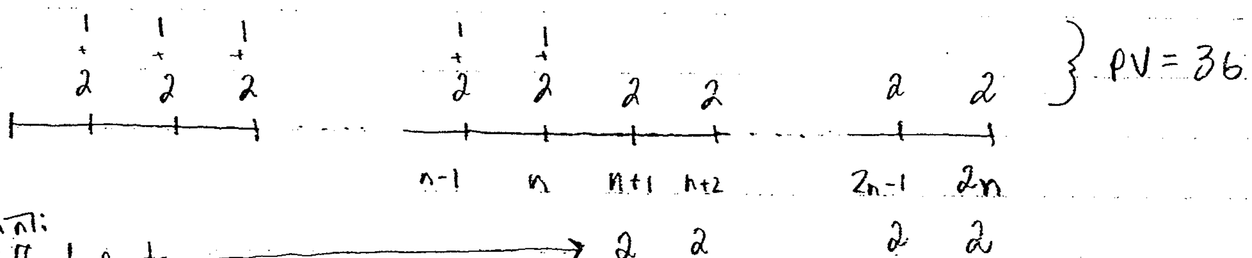
$\Rightarrow 1.8061a + 1.6386b = 10\,000$ at $t=31 \Rightarrow \dfrac{a\cdot(1.03)^{31}}{b\cdot(1.025)^{31}} = 2 \Rightarrow$

$2.5a = 2\cdot b\cdot 2.15 = 4.3b \Rightarrow a = \dfrac{4.3b}{2.5} \Rightarrow 1.8061\left(\dfrac{4.3b}{2.5}\right) + 1.6386b = 10$

$4.7451b = 10\,000$ $b = 2107.44 \Rightarrow a = 3624.74$

$\Rightarrow$ FV at $t=10$: $3624.74\cdot(1.03)^{10} + 2107.44\cdot(1.025)^{10} = \boxed{7569.09}$

(8.)



$2\cdot{}_{n|}a_{\overline{n}|}$:
n-year deffered pymts

This suggests that $(1+2)\cdot a_{\overline{n}|} + 2\cdot{}_{n|}a_{\overline{n}|}$ is also equal to 36

$\Rightarrow (1+2)\cdot a_{\overline{n}|} + 6 = 36 \Rightarrow 3\cdot a_{\overline{n}|} = 30 \Rightarrow a_{\overline{n}|} = 10$

$\Rightarrow 2a_{\overline{n}|}\cdot v^n = 6 \Rightarrow 2(10)\cdot v^n = 6 \Rightarrow v^n = \dfrac{6}{20}$

$\Rightarrow a_{\overline{n}|} = \dfrac{1-v^n}{i} = \dfrac{1-\frac{6}{20}}{i} = 10$ $10i = \dfrac{14}{20}$ $\boxed{i = .07}$

(9.) Dollar amount of interest earned by Cory: $4 \cdot X \cdot i = 4 \cdot X(.10) = .4X$

For Julie: $AV_8 - AV_4$ = dollar amount of interest earned btw $t=4$ and $t=8$.

$$AV_4 = X \cdot \exp\left[\int_0^4 \frac{2t}{t^2+k}\,dt\right] = X \cdot \exp\left[\ln(t^2+k)\Big|_0^4\right] = X \cdot \exp\left[\ln(16+k) - \ln(0+k)\right]$$

$$= X \cdot \left(e^{\ln(16+k)} \cdot e^{\ln(k)}\right) = X \cdot \frac{(k+16)}{k}$$

$$AV_8 = X \exp\left[\int_0^8 \frac{2t}{t^2+k}\,dt\right] = X \cdot \frac{(k+8^2)}{k} = X\,\frac{(k+64)}{k}$$

$$\text{Interest Earned by Julie} = X \cdot \left[\frac{(k+64)}{k} - \frac{(k+16)}{k}\right]$$

$$\Rightarrow X \cdot \left[\left(\frac{k+64}{k}\right) - \left(\frac{k+16}{k}\right)\right] = .4X \quad \text{so} \quad .4 = \frac{k+64}{k} - \frac{(k+16)}{k} \Rightarrow$$

$$.4k = k+64-k-16 \qquad \boxed{K = 120}$$

(10.)

6000	?		9725.78
0	1	2	3

$$\Rightarrow \left[6000(1+1\cdot(.07)) + X\right] \cdot e^{.08(1)} \cdot \frac{1}{(1-.09)^1} = 9725.78$$

$$\Rightarrow 7642.53 + 1.19042 \cdot X = 9725.78 \quad \Rightarrow \boxed{X = 1750}$$